1972

A static analysis of a synchronous transformer.

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Massachusetts Institute of Technology

http://hdl.handle.net/10945/16365

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by

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(1964)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

OCEAN ENGINEER

AND THE DEGREE OF

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

A STATIC ANALYSIS OF A SYNCHRONOUS TRANSFORMER

by

John Paul Kelsey

Submitted to the Department of Ocean Engineering on May 12, 1972, in partial fulfillment of the requirements for the degrees of Ocean Engineer and Master of Science in Electrical Engineering

Abstract

The use of superconductors in the field windings of large synchronous machines displaces iron as a major element in the production of the magnetic field and promises smaller, less costly machines for a given rating. The possibility of replacing the iron in a transformer bank with a rotating superconducting field is discussed. By placing the primary and secondary three phase windings in separate armature windings built in epoxy and fiberglass concentric shells around the superconducting, free-spinning field and allowing the angular alignment of one of the armatures to be varied, a unique transformer is devised. Unique features of the synchronous transformer are: the ability to separately control real and reactive power flow, thus providing power factor correction while simultaneously acting as a transformer, and the ability to transform rated real power between two power systems that are out of time phase, with zero (or at a specified level within a range) reactive power flow.

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ACKNOWLEDGMENTS

The author wishes to express his gratitude to Professor J. L. Smith, Jr., Department of Mechanical Engineering, M.I.T., who originally conceived the idea of the synchronous transformer, to Professor J. L. Kirtley, Jr., Department of Electrical Engineering, M.I.T., who supervised and assisted my efforts on this thesis, and to the United States Navy and Edison Electric Institute, who provided financial support.

Table of Contents

List of Illustrations

List of Tables

INTRODUCTION

Synchronous machines are commonplace in the modern world and the theory of their operation is well developed. The application of superconducting technology to the field windings of synchronous machines is a new and growing area of research [1]. It is expected that superconducting machines with ratings of 1000 KVA and greater may have as much as an order of magnitude reduction in weight and size over a conventional machine of the same rating with only about 0.1 per cent of machine rating required to power the refrigeration unit of the superconductor [2].

Recent work at M.I.T. anticipates that it may be possible to manufacture and ship synchronous generators with superconducting field windings with ratings as high as 10,000 MVA and that construction and operation of these machines may be more economical than current conventional machines [3].

These economic advantages are brought about in part by the very high flux densities with minimal power dissipation achievable with superconductors, which makes it possible to eliminate the magnetic circuit except possibly as an external shield. Thus an expensive magnetic-iron rotor forging and stator teeth may be eliminated, which in turn reduces insulation requirements (or allows an increase in voltage rating) and at the same time increases the amount of space that may

be filled with armature conductors. Elimination of most of the iron from the machine reduces per unit reactances which may improve stability.

References [2] and [4] derive expressions for electrical parameters of synchronous machines without magnetic shielding. References [5] and [6] extend these equations to machines with an exterior iron shield.

In a superconducting alternator the superconducting field winding displaces iron as a major element in the production of the magnetic field. It might be asked if this can be done for other devices, specifically, is it feasible to replace the iron in a transformer bank with a rotating superconducting field? Such a device, hereafter called a synchronous transformer, would have two armature windings (one for each of two sets of three phase terminals) of the form contemplated for a superconducting alternator. It would also have a superconducting field winding, which is free spinning as does the field winding of a synchronous condenser.

Electric energy is transferred from one armature to the other by the interaction of the two armature flux waves with the field winding flux wave: one interaction being similar to that of a motor, the other like a generator.

As it will be shown, this proposed device has similarities with conventional transformers, but it has important differences too. For example, if the angular alignment of

one of the two armatures is made variable then the device has two "handles" on power flow: armature position and main field current. This contrasts with the availability of only one "handle" (tap changing) with conventional transformers. This thesis will show that these two control variables will enable the synchronous transformer to control separately both real and reactive power flow. Thus it can act as a power factor correction device while simultaneously acting as a transformer. Additionally, it can transform rated real power between two power system buses that are out of time phase with no reactive power flow or within a range of desired reactive power levels.

Such a device could be constructed by embedding the armature windings in epoxy and fiberglass to a fiberglass (or other non-magnetic material) pipe for structural strength [7,6]. Two such armature windings would be required. The axis of the outer armature would be adjustable so that it could be rotated about the longitudinal axis of the rotor (and thus also about the longitudinal axis of the inner armature) by an external control, but it would not be free to spin. The returning flux path and shielding would be provided by a concentric iron shell external to the two armature shells. Figure 1 illustrates the physical configuration of the synchronous transformer. Figure 2 illustrates the electrical configuration.

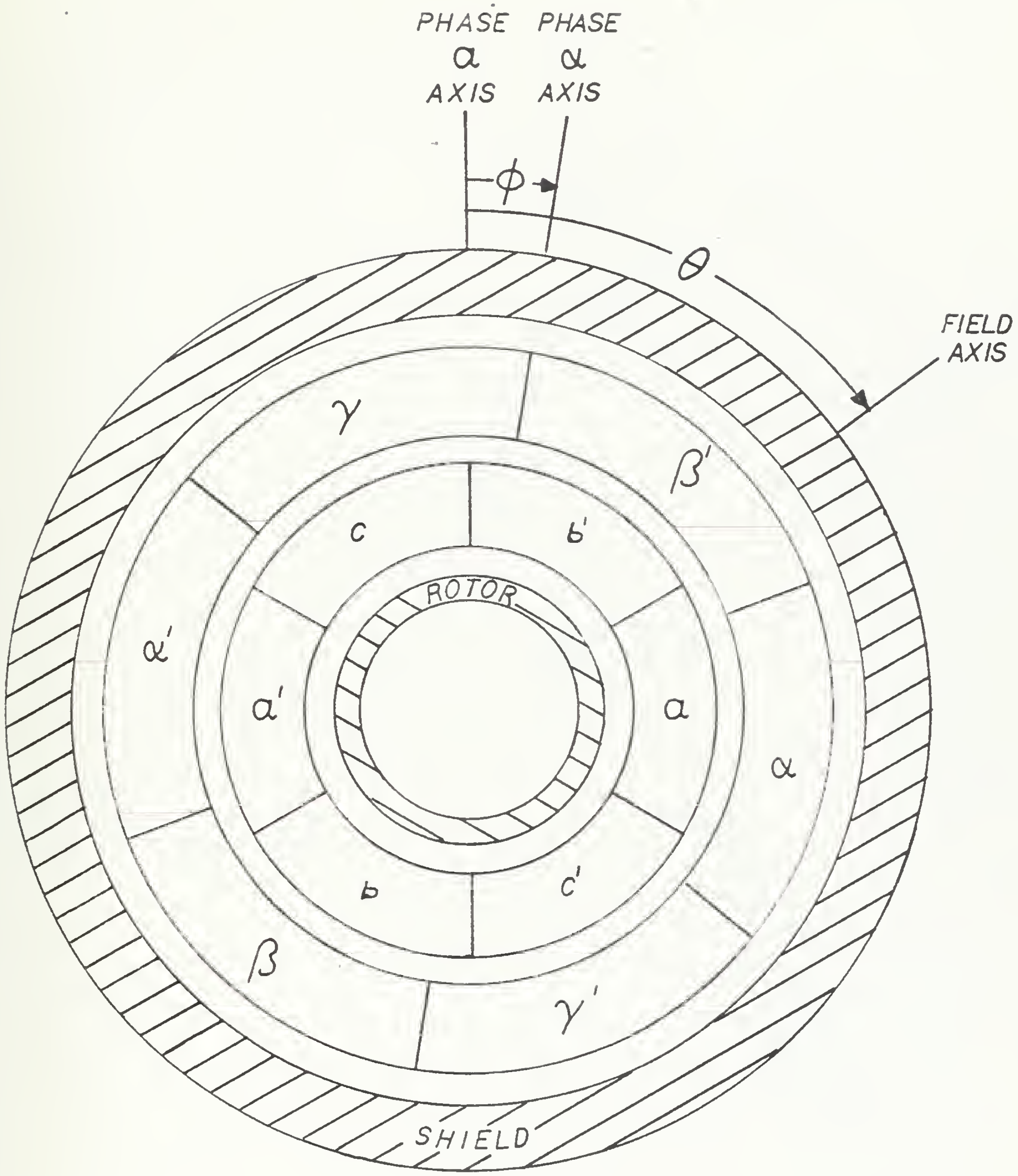

Figure 1

Physical Configuration of the Synchronous Transformer, End View

Figure 2

Electrical Configuration

Armature 2 is Shown Rotated by an Angle ϕ from Armature 1

There is a growing interest in the use of superconducting machinery for marine propulsion. In 1966 Sibley et al [8] estimated some potential advantages of superconducting electrical machinery over conventional propulsion plants. In a later work Laine [9] considered the possibility of using a superconducting transmission link in a destroyer-type ship. Matjasko [10] investigated a control system for a destroyer propelled by superconducting electric machinery. Greene [11] examined transmission systems including a cycloconverter scheme between a synchronous generator and an 18,000 SHP (13.5 MW) synchronous motor driving a propeller.

While in its present configuration it would seem that the synchronous transformer would perhaps be more useful to the electric power industry than to marine propulsion, it is, however, similar in configuration to a class of dual armature superconducting machines which may have great potential as an electric propulsion ship system. This class of AC machines may have the control characteristics of a DC machine system but with the additional advantage that a solid state switching circuit will replace the bulky cycloconverter. This class of dual armature machines will not be discussed further here as they are the proprietary information of the Cryogenic Engineering Laboratory at M.I.T.

Table 1

GLOSSARY OF TERMS

Note: Unless otherwise noted, the following subscripts will serve as modifiers to the symbols.

Subscripts

a	phase a, armature 1
b	phase b, armature 1
c	phase c, armature 1
α	phase α, armature 2
β	phase β, armature 2
γ	phase γ, armature 2
d	direct axis, armature 1
q	quadrature axis, armature 1
δ	direct axis, armature 2
ξ	quadrature axis, armature 2
f	field
o	zero sequence, armature 1
Ω	zero sequence, armature 2
P_1	phase variables, armature 1
P_2	pnase variables, armature 2
R_1	rotor variables, armature 1
R_2	rotor variables, armature 2
L_1	armature 1 to field
L_2	armature 2 to field
t_1	equivalent at the terminals, armature 1
t_2	equivalent at the terminals, armature 2

Glossary of Terms
(continued)

Symbols

i	current
v	voltage
λ	winding flux
L	inductance
θ	angular position of the rotor from the axis of phase a, armature 1, positive in direction of rotation
ϕ	angular position of the axis of phase α, armature 2, from the axis of phase a, armature 1, positive in the direction of rotation
M	maximum value of mutual inductance between phase a, armature 1, and phase α, armature 2 (i.e., $L_{a\alpha}$ at $\phi = 0$)
L_1	maximum value of mutual inductance between phase a, armature 1, and the field (i.e., L_{af} at $\theta = 0$)
L_2	maximum value of mutual inductance between phase α, armature 2, and the field (i.e., L_{af} at $\theta = \phi$)
δ_a	spatial angle from the rotor axis to the armature flux wave of armature 1, positive in the direction of rotation.
δ_o	spatial angle from the armature flux wave of armature 1 to the armature flux wave of armature 2, positive in the direction of rotation
V_1	terminal voltage of armature 1 in per unit
V_2	terminal voltage of armature 2 in per unit
x	per unit reactance
E	air gap voltage

RESULTS OF A STATIC ANALYSIS

For any winding of a synchronous machine the voltage at the terminals is equal to the product of the winding resistance and current through the winding plus the time rate of change of the winding flux. For any winding the flux is the summation of the self inductance times the current through the winding and the product of mutual inductances with other windings (which may depend on rotor angular position) and the currents through those windings.

Thus:

$$V_m = r_m i_m + \frac{d\lambda_m}{dt} \tag{1}$$

$$\lambda_m = \sum_n \lambda_{nm} i_n \tag{2}$$

Losses in large synchronous machines are less than one per cent of machine rating and the assumption of a lossless model does not have much effect on the static analysis. Therefore one can assume that winding resistances are neglible yielding:

$$V_m = \frac{d\lambda_m}{dt} \tag{3}$$

The machine flux current relationships may be expressed by:

$$
\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_\alpha \\ \lambda_\beta \\ \lambda_\gamma \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_a & L_{ab} & L_{ac} & L_{a\alpha} & L_{a\beta} & L_{a\gamma} & L_{af} \\ L_{ab} & L_b & L_{bc} & L_{b\alpha} & L_{b\beta} & L_{b\gamma} & L_{bf} \\ L_{ac} & L_{bc} & L_c & L_{c\alpha} & L_{c\beta} & L_{c\gamma} & L_{cf} \\ L_{a\alpha} & L_{b\alpha} & L_{c\alpha} & L_\alpha & L_{\alpha\beta} & L_{\alpha\gamma} & L_{\alpha f} \\ L_{a\beta} & L_{b\beta} & L_{c\beta} & L_{\alpha\beta} & L_\beta & L_{\beta\gamma} & L_{\beta f} \\ L_{a\gamma} & L_{b\gamma} & L_{c\gamma} & L_{\alpha\gamma} & L_{\beta\gamma} & L_\gamma & L_{\gamma f} \\ L_{af} & L_{bf} & L_{cf} & L_{\alpha f} & L_{\beta f} & L_{\gamma f} & L_f \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_\alpha \\ i_\beta \\ i_\gamma \\ i_f \end{bmatrix} \tag{4}
$$

Because of fixed geometry the following relations apply:

$$L_a = L_b = L_c \tag{5}$$

$$L_\alpha = L_\beta = L_\gamma \tag{6}$$

$$L_{ab} = L_{ac} = L_{bc} \tag{7}$$

$$L_{\alpha\beta} = L_{\beta\gamma} = L_{\alpha\gamma} \tag{8}$$

It is important to note that the geometry between armatures is not fixed since armature 2 may be rotated and since phase axes are 120° apart.

$$L_{a\alpha} \neq L_{a\beta} \neq L_{a\gamma} \tag{9}$$

It is assumed that all rotor to armature mutual inductances vary with the cosine of rotor position, with the maximum values of L_{af} and $L_{\alpha f}$ when the rotor axis aligns with the respective phase axes of armature 1 and armature 2. This is the same as neglecting higher order space harmonics in the

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_\alpha \\ \lambda_\beta \\ \lambda_\gamma \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_a & L_{ab} & L_{ab} & M\cos\phi & M\cos(\frac{2\pi}{3}+\phi) & M\cos(\frac{2\pi}{3}-\phi) & L_1\cos\theta \\ L_{ab} & L_a & L_{ab} & M\cos(\frac{2\pi}{3}-\phi) & M\cos\phi & M\cos(\frac{2\pi}{3}+\phi) & L_1\cos(\frac{2\pi}{3}-\theta) \\ L_{ab} & L_{ab} & L_a & M\cos(\frac{2\pi}{3}+\phi) & M\cos(\frac{2\pi}{3}-\phi) & M\cos\phi & L_1\cos(\frac{2\pi}{3}+\theta) \\ M\cos\phi & M\cos(\frac{2\pi}{3}-\phi) & M\cos(\frac{2\pi}{3}+\phi) & L_\alpha & L_{\alpha\beta} & L_{\alpha\beta} & L_2\cos(\theta-\phi) \\ M\cos(\frac{2\pi}{3}+\phi) & M\cos\phi & M\cos(\frac{2\pi}{3}-\phi) & L_{\alpha\beta} & L_\alpha & L_{\alpha\beta} & L_2\cos(\frac{2\pi}{3}-\theta+\phi) \\ M\cos(\frac{2\pi}{3}-\phi) & M\cos(\frac{2\pi}{3}+\phi) & M\cos\phi & L_{\alpha\beta} & L_{\alpha\beta} & L_\alpha & L_2\cos(\frac{2\pi}{3}+\theta-\phi) \\ L_1\cos\theta & L_1\cos(\frac{2\pi}{3}-\theta) & L_1\cos(\frac{2\pi}{3}+\theta) & L_2\cos(\theta-\phi) & L_2\cos(\frac{2\pi}{3}-\theta+\phi) & L_2\cos(\frac{2\pi}{3}+\theta-\phi) & L_f \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_\alpha \\ i_\beta \\ i_\gamma \\ i_f \end{bmatrix} \tag{10}$$

rotor air gap flux and assuming a non-salient rotor. It is further assumed that armature 2 to armature 1 mutual inductances vary as the cosine of the relative angular position of the respective phase axes with the maximum value of $L_{a\alpha}$ when the armature displacement angle, ϕ, equals zero. This value $L_{a\alpha}$ (maximum) will be denoted by M. This assumption is equivalent to neglecting higher order space harmonics in the air gap flux between armatures. Higher order space harmonics die away more rapidly than the fundamental. Since this is an air gap machine, disregarding higher order space harmonics in the air gaps will introduce only a small error [6]. Substitution into equation (4) yields equation (10).

Equation (3) gives:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_\alpha \\ V_\beta \\ V_\gamma \\ V_f \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_\alpha \\ \lambda_\beta \\ \lambda_\gamma \\ \lambda_f \end{bmatrix} \tag{11}$$

At this point it is convenient to transform equation (11) into the rotating coordinate system of the rotor. The subscripts d and q correspond to quantities of stator 1,

$$\begin{bmatrix} v_d \\ v_q \\ v_o \\ v_\delta \\ v_\xi \\ v_\Omega \\ v_f \end{bmatrix} = \begin{bmatrix} (L_a - L_{ab})\frac{d}{dt} & -(L_a - L_{ab})\frac{d\theta}{dt} & 0 & \frac{3}{2}M(\cos\phi\frac{d}{dt} - \sin\phi\frac{d\theta}{dt}) & \frac{3}{2}M(-\sin\phi\frac{d}{dt} - \cos\phi\frac{d\theta}{dt}) & 0 & \sqrt{\frac{3}{2}}\, L_1 \frac{d}{dt} \\ (L_a - L_{ab})\frac{d\theta}{dt} & (L_a - L_{ab})\frac{d}{dt} & 0 & \frac{3}{2}M(\sin\phi\frac{d}{dt} + \cos\phi\frac{d\theta}{dt}) & \frac{3}{2}M(\cos\phi\frac{d}{dt} - \sin\phi\frac{d\theta}{dt}) & 0 & \sqrt{\frac{3}{2}}\, L_i \frac{d\theta}{dt} \\ 0 & 0 & (L_a + 2L_{ab})\frac{d}{dt} & 0 & 0 & 0 & 0 \\ \frac{3}{2}M(\cos\phi\frac{d}{dt} + \sin\phi\frac{d\theta}{dt}) & \frac{3}{2}M(\sin\phi\frac{d}{dt} - \cos\phi\frac{d\theta}{dt}) & 0 & (L_\alpha - L_{\alpha\beta})\frac{d}{dt} & -(L_\alpha - L_{\alpha\beta})\frac{d\theta}{dt} & 0 & \sqrt{\frac{3}{2}}\, L_1 \frac{d}{dt} \\ \frac{3}{2}M(-\sin\phi\frac{d}{dt} + \cos\phi\frac{d\theta}{dt}) & \frac{3}{2}M(\cos\phi\frac{d}{dt} + \sin\phi\frac{d\theta}{dt}) & 0 & (L_\alpha - L_{\alpha\beta})\frac{d\theta}{dt} & (L_\alpha - L_{\alpha\beta})\frac{d}{dt} & 0 & \sqrt{\frac{3}{2}}\, L_2 \frac{d\theta}{dt} \\ 0 & 0 & 0 & 0 & 0 & (L_a + 2L_{\alpha\beta})\frac{d}{dt} & 0 \\ \sqrt{\frac{3}{2}}\, L_1 \frac{d}{dt} & 0 & 0 & \sqrt{\frac{3}{2}}\, L_2 \frac{d}{dt} & 0 & 0 & L_f \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_\delta \\ i_\xi \\ i_\Omega \\ i_f \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \\ \lambda_\delta \\ \lambda_\xi \\ \lambda_\Omega \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_a - L_{ab} & 0 & 0 & \frac{3}{2} M \cos\phi & -\frac{3}{2} M \sin\phi & 0 & \sqrt{\frac{3}{2}} L_1 \\ 0 & L_a - L_{ab} & L_a + 2L_{ab} & \frac{3}{2} M \sin\phi & \frac{3}{2} M \cos\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{2} M \cos\phi & \frac{3}{2} M \sin\phi & 0 & L_\alpha - L_{\alpha\beta} & 0 & 0 & \sqrt{\frac{3}{2}} L_2 \\ -\frac{3}{2} M \sin\phi & \frac{3}{2} M \cos\phi & 0 & 0 & L_\alpha - L_{\alpha\beta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_\alpha + 2L_{\alpha\beta} & 0 \\ \sqrt{\frac{3}{2}} L_1 & 0 & 0 & \sqrt{\frac{3}{2}} L_2 & 0 & 0 & L_f \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_\delta \\ i_\xi \\ i_\Omega \\ i_f \end{bmatrix} \tag{13}$$

rotating with the direct and quadrature axis of the field. The subscript, o, corresponds to zero sequence quantities of stator 1, that is in phase in all three windings of stator 1. The corresponding subscripts for stator 2 are δ, ξ, and Ω. The details of this transformation are given in Appendix A.

The result of transforming to rotor coordinates yields equations (12) and (13).

By comparison of equations (12) and (13) the following relationships can be recognized:

$$V_d = \frac{d}{dt}(\lambda_d) - \omega\lambda_q \tag{14}$$

$$V_q = \frac{d}{dt}(\lambda_q) + \omega\lambda_d \tag{15}$$

$$V_\delta = \frac{d}{dt}(\lambda_\delta) - \omega\lambda_\xi \tag{16}$$

$$V_\xi = \frac{d}{dt}(\lambda_\xi) + \omega\lambda_\delta \tag{17}$$

$$V_f = \frac{d}{dt}\lambda_f \tag{18}$$

It can be seen that zero sequence variables can be eliminated from equations (12) and (13) since they are completely uncoupled from other variables.

For ease in notation the following definitions are introduced:

$$L_E = L_a - L_{ab} \tag{19}$$

$$L_\varepsilon = L_\alpha - L_{\alpha\xi} \tag{20}$$

Thus equation (13) becomes:

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_\delta \\ \lambda_\xi \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_E & 0 & \frac{3}{2} M \cos\phi & -\frac{3}{2} M \sin\phi & \sqrt{\frac{3}{2}} L_1 \\ 0 & L_E & \frac{3}{2} M \sin\phi & \frac{3}{2} M \cos\phi & 0 \\ \frac{3}{2} M \cos\phi & \frac{3}{2} M \sin\phi & L_\varepsilon & 0 & \sqrt{\frac{3}{2}} L_2 \\ -\frac{3}{2} M \sin\phi & \frac{3}{2} M \cos\phi & 0 & L_\varepsilon & 0 \\ \sqrt{\frac{3}{2}} L_1 & 0 & \sqrt{\frac{3}{2}} L_2 & 0 & L_f \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_\delta \\ i_\xi \\ i_f \end{bmatrix} \tag{21}$$

Now that the basic relations governing the synchronous transformer have been developed, the operation of the transformer between two buses will be looked at. Motor sign convention will be observed throughout, that is, real power input to the machine from either system (bus) is defined as positive and lagging current determines the positive direction for reactive power.

Since the flux-current relationships have been transformed into rotor d-q coordinates, the following definitions of armature equivalent flux vectors are introduced:

$$\lambda_{t1} = \sqrt{\lambda_d^{\,2} + \lambda_q^{\,2}} \tag{22}$$

$$\lambda_{t2} = \sqrt{\lambda_\delta^{\,2} + \lambda_\xi^{\,2}} \tag{23}$$

Similarly, the equivalent armature voltage vectors are likewise defined:

$$V_{t1} = \sqrt{V_d^{\,2} + V_q^{\,2}} \tag{24}$$

$$V_{t2} = \sqrt{V_\delta^{\,2} + V_\xi^{\,2}} \tag{25}$$

The angle δ_a is defined as the spatial angle from the rotor axis to the armature flux of armature 1, positive in the direction of rotation. The angle δ_o is defined as the spatial angle from the armature flux wave of armature 1 to the armature flux wave of armature 2, positive in the direction of rotation. Figure 3 illustrates these sign conventions.

Alternatively, since the voltage vectors are a ninety degree rotation of the armature flux wave vectors, these same definitions can be viewed as applying equally between the air gap voltage, terminal voltage of armature 1 and terminal voltage of armature 2. Figure 4 illustrates the sign conventions of angles δ_a and δ_o in terms of the voltage vectors.

The following definitions follow directly:

$$V_d = - V_{t1} \sin\delta_a \tag{26}$$

$$V_q = V_{t1} \cos\delta_a \tag{27}$$

$$V_\delta = -V_{t2} \sin(\delta_a + \delta_o) \tag{28}$$

$$V_\xi = V_{t2} \cos(\delta_a + \delta_o) \tag{29}$$

If the restriction to strict steady state operation is assumed, then time derivative terms become zero. Equations (14) through (17) become:

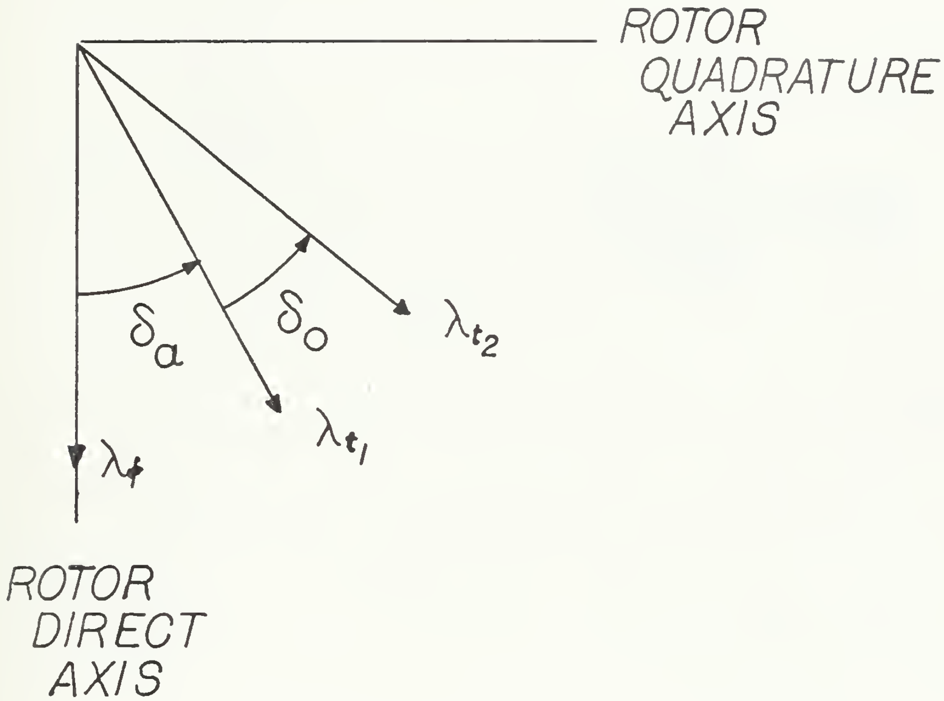

Figure 3

Illustration of Sign Conventions for Spatial Angles δ_a and δ_o

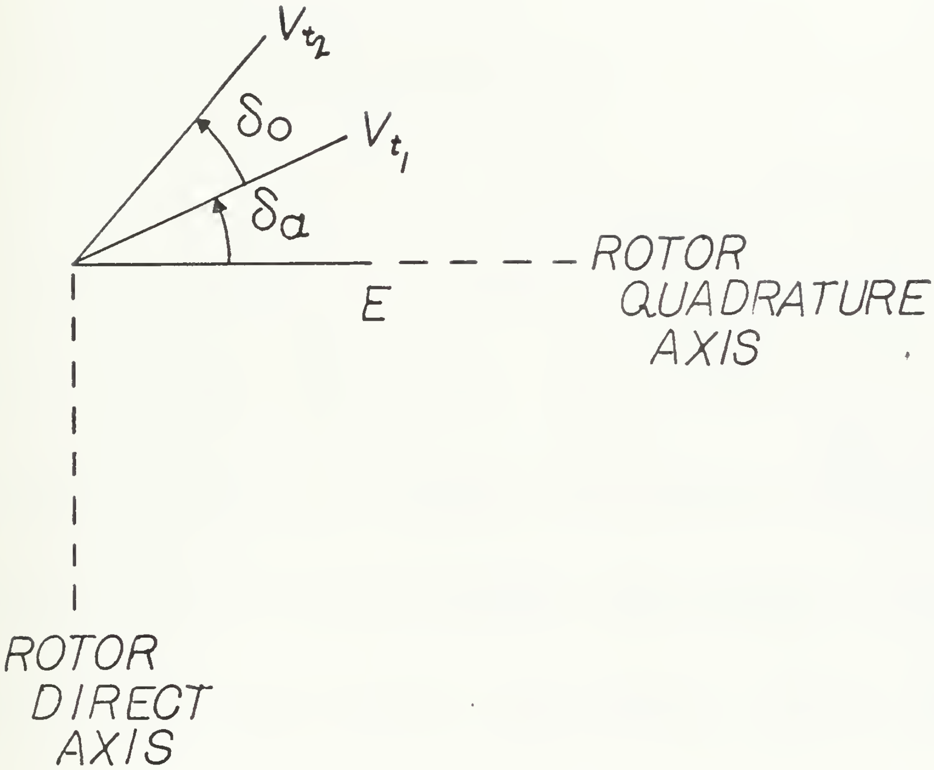

Figure 4

Illustration of Sign Conventions for Spatial Angles δ_a and δ_o in Terms of Voltage Vectors

$$V_d = -\omega\lambda_q \tag{30}$$

$$V_q = \omega\lambda_d \tag{31}$$

$$V_\delta = -\omega\lambda_\xi \tag{32}$$

$$V_\xi = \omega\lambda_\delta \tag{33}$$

Substituting of equations (21) through (29) into equations (30) through (33) yields:

$$V_{t1}\cos\delta_a = -\omega(L_E i_q + \frac{3}{2} M \sin\phi i_\delta + \frac{3}{2} M \cos\phi i_\xi) \tag{34}$$

$$V_{t1}\sin\delta_a = \omega(L_E i_d + \frac{3}{2} M \cos\phi i_\delta - \frac{3}{2} M \sin\phi i_\xi + \sqrt{\frac{3}{2}} L_1 i_f) \tag{35}$$

$$V_{t2}\cos(\delta_a + \delta_o) = -\omega(-\frac{3}{2} M \sin\phi i_d + \frac{3}{2} M \cos\phi i_q + L_\varepsilon i_\xi) \tag{36}$$

$$V_{t2}\sin(\delta_a+\delta_o) = \omega(\frac{3}{2} M \cos\phi i_d + \frac{3}{2} M \sin\phi i_q + L_\varepsilon i_\delta + \sqrt{\frac{3}{2}} L_2 i_f \tag{37}$$

Since the rotor is free spinning there will be no electrical-mechanical power transfer from the rotor. If we ignore all losses (friction, windage, copper losses, etc.) then the net power of the synchronous transformer will be zero.

$$0 = V_d i_d + V_q i_q + V_\delta i_\delta + V_\xi i_\xi \tag{38}$$

This is not to say that the device cannot absorb power from one bus under the lossless assumption, but rather

equation (38) tells us that real power into either armature must go out the other armature.

Substitution of equations((14) through (17) with the steady state asumption $d/dt \equiv 0$ and equation (21) into equation (38) yields:

$$0 = \omega L_1 i_q + \omega L_2 i_\xi \tag{39}$$

Equations (34) through (37) and equation (39) are the governing equations for the synchronous transformer operating between two buses. Independent variables are i_f, ϕ, V_1 and V_2. Dependent variables are i_d, i_δ, i_q, i_ξ, δ_a, and δ_o. Thus we have five equations and six unknowns. The method of solution used is to assign values of δ_o for a sufficiently wide range and at each value of δ_o solve the resulting equations which then have only five unknowns. However, before proceeding to the solutions of these equations it is convenient to put them into the per unit system. Details of this are in Appendix B.

The governing equations for the synchronous transformer operating between two buses in per unit are:

$$V_1 \sin\delta_a = x_d i_q + x_m i_\delta \sin\phi + x_m i_\xi \cos\phi \tag{40}$$

$$V_1 \cos\delta_a = x_d i_d + x_m i_\delta \cos\phi - x_m i_\xi \sin\phi + x_{L1} i_f \tag{41}$$

$$V_2 \sin(\delta_a+\delta_o) = - x_m i_d \sin\phi + x_m i_q \cos\phi + x_\delta i_\xi \tag{42}$$

$$V_2 \cos(\delta_a+\delta_o) = x_m i_d \cos\phi + x_m i_q \sin\phi + x_\delta i_\delta + x_{L2} i_f \tag{43}$$

$$0 = x_{L1} i_q + x_{L2} i_\xi \tag{44}$$

In order to use Gauss-Seidel computer aided iterative solution these equations were rewritten as follows:

$$\delta_a = \sin^{-1}\left[\frac{x_d i_q + x_m i_\delta \sin\phi + x_m i_\xi \cos\phi}{V_1}\right] \tag{45}$$

$$i_d = \frac{V_1 \cos\delta_a - x_m i_\delta \cos\phi + x_m i_\xi \sin\phi - x_{L1} i_f}{x_d} \tag{46}$$

$$i_\xi = \frac{V_2 \sin(\delta_a + \delta_o) + x_m i_d \sin\phi - x_m i_q \cos\phi}{x_\delta} \tag{47}$$

$$i_\delta = \frac{V_2 \cos(\delta_a + \delta_o) - x_m i_d \cos\phi - x_m i_q \sin\phi - x_{L2} i_f}{x_\delta} \tag{48}$$

$$i_q = -\frac{x_{L2} i_\xi}{x_{L1}} \tag{49}$$

Appendix (C) discussed the Gauss-Seidel iterative solution. Reference 12 discusses this and other applicable methods in great detail.

Reasonable values of per-unit inductances for a synchronous machine with a superconducting field winding were chosen [6, 13]. The assigned inductances are listed in Table 2.

Table 2

PER-UNIT REACTANCES USED IN STATIC ANALYSIS

Armature 1 Synchronous Reactance	$x_d = 0.5$ p.u.
Armature 2 Synchronous Reactance	$x_\delta = 0.5$ p.u.
Mutual Reactance Between Armatures	$x_m = 0.4$ p.u.
Armature 1 to Field Resistance	$x_{L1} = 1.0$ p.u.
Armature 2 to Field Reactance	$x_{L2} = 1.0$ p.u.

A computer program was prepared to solve simultaneous equations (45) through (49) by the Gauss-Seidel iterative method for a range of armature displacement angles, ϕ, from -90° to +90° and a range of spatial angles between armature voltage vectors, δ_o, from 0° to 90° and for different levels of field current. In all cases reactances from Table 2 and one per unit terminal voltages were used. For each level of field current and each choice of angles ϕ and δ_o the computer calculated and printed values of direct and quadratic axis currents and voltages for both armatures, real and reactive power for both armatures, and the spatial angle between the air gap voltage vector and the voltage vector of armature 1. This produced a wealth of data from which Figures 5 through 10 were extracted.

It is important to note here that while the angles δ_a and δ_o are spatial angles relating the voltage vectors to the rotor, the angle ϕ is a physical angle by which the armatures are displaced. Thus the time phase angle between the power

systems attached to the armatures is a combination of their physical and electrical angular difference, $\delta_o + \phi$. Since we are not interested in internal angles to characterize the device this power system time phase angle will be the significant angle used to characterize the device.

Since we are primarily interested in the synchronous transformer to transfer power, the solutions yielding real power equal to one per unit were collated and graphed in Figures 5 through 7. Figures 8 through 10 were constructed from Figures 6 and 7. Figure 8 was obtained by cross plotting the armature displacement angle, ϕ, from Figure 6 corresponding to the zero crossings of reactive power flow from Figure 7. Thus Figure 8 gives the setting points for armature displacement angle, ϕ, and field current, I_f, for a range of power system time phase differences in which one per unit real power can be transferred with no reactive power. Figures 9 and 10 yield similar results for reactive power flow equal to plus and minus 0.1 per unit.

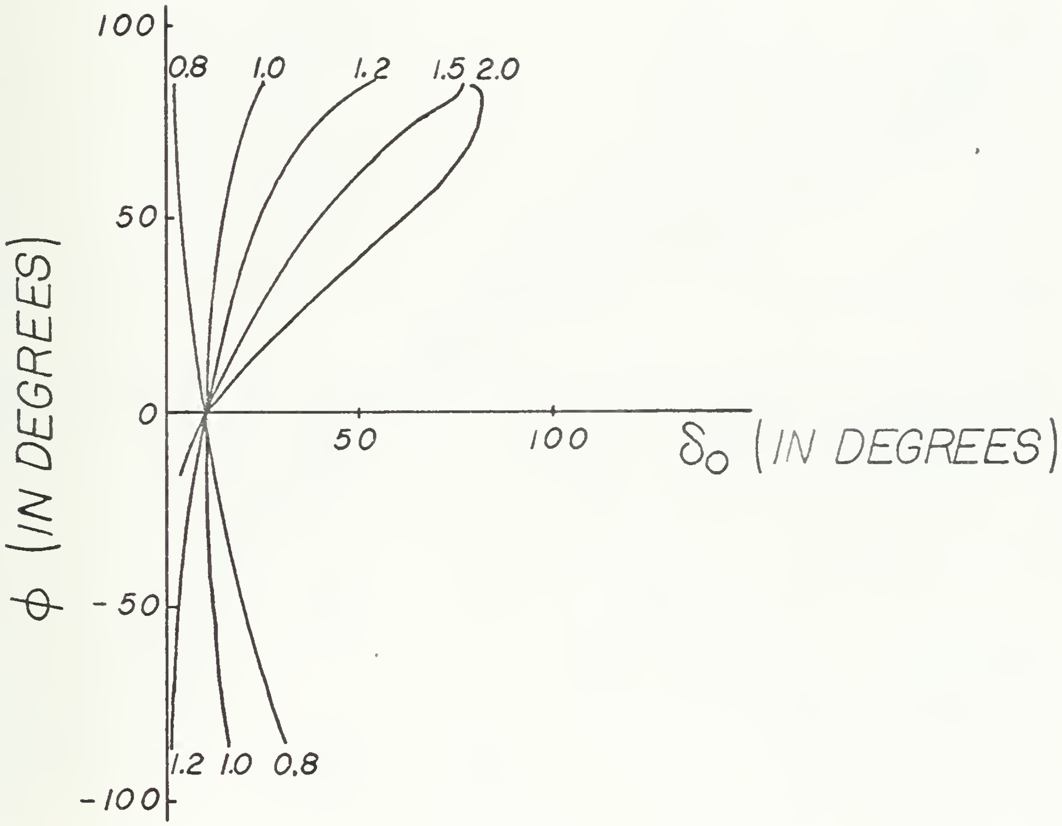

Figure 5

Curves of armature displacement angle, ϕ, versus spatial angle between armature voltage vectors for one per unit real power transfer and various levels of field current, I_f

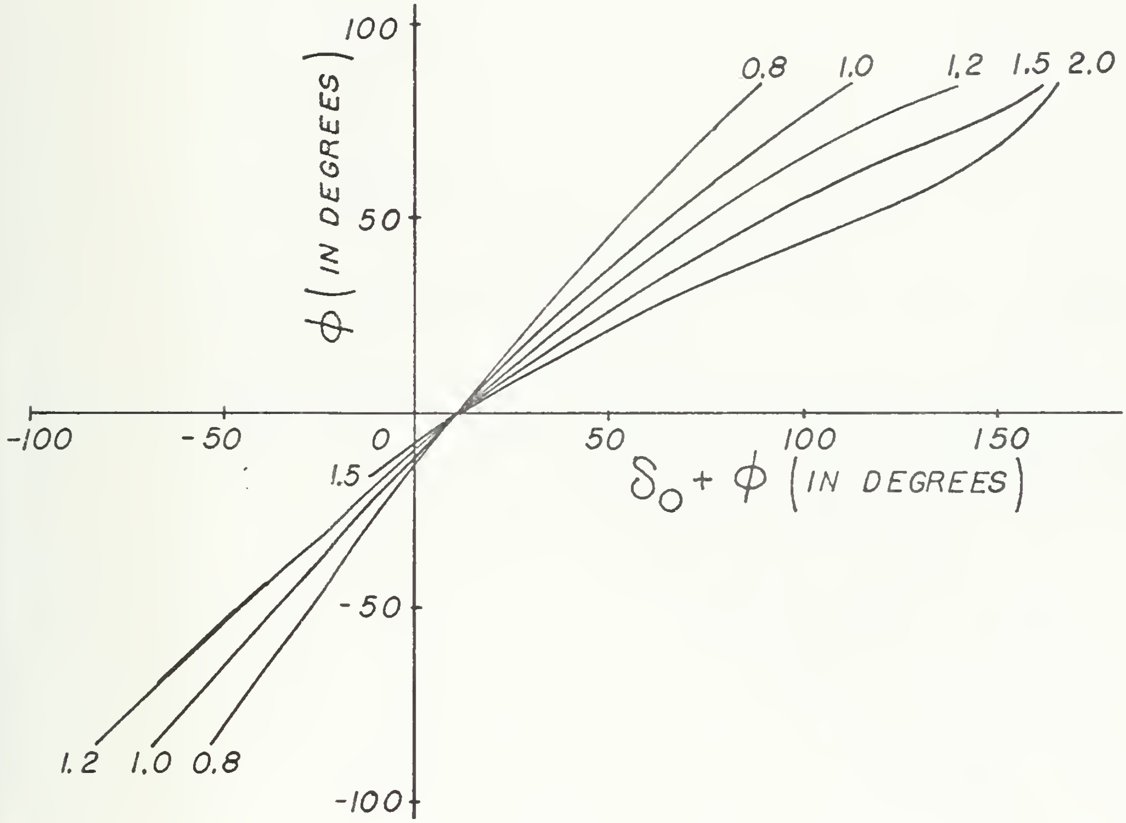

Figure 6

Curves of armature displacement angle, ϕ, versus power system time phase angle difference, $\delta_o + \phi$, for one per unit real power transfer and various levels of field current, I_f

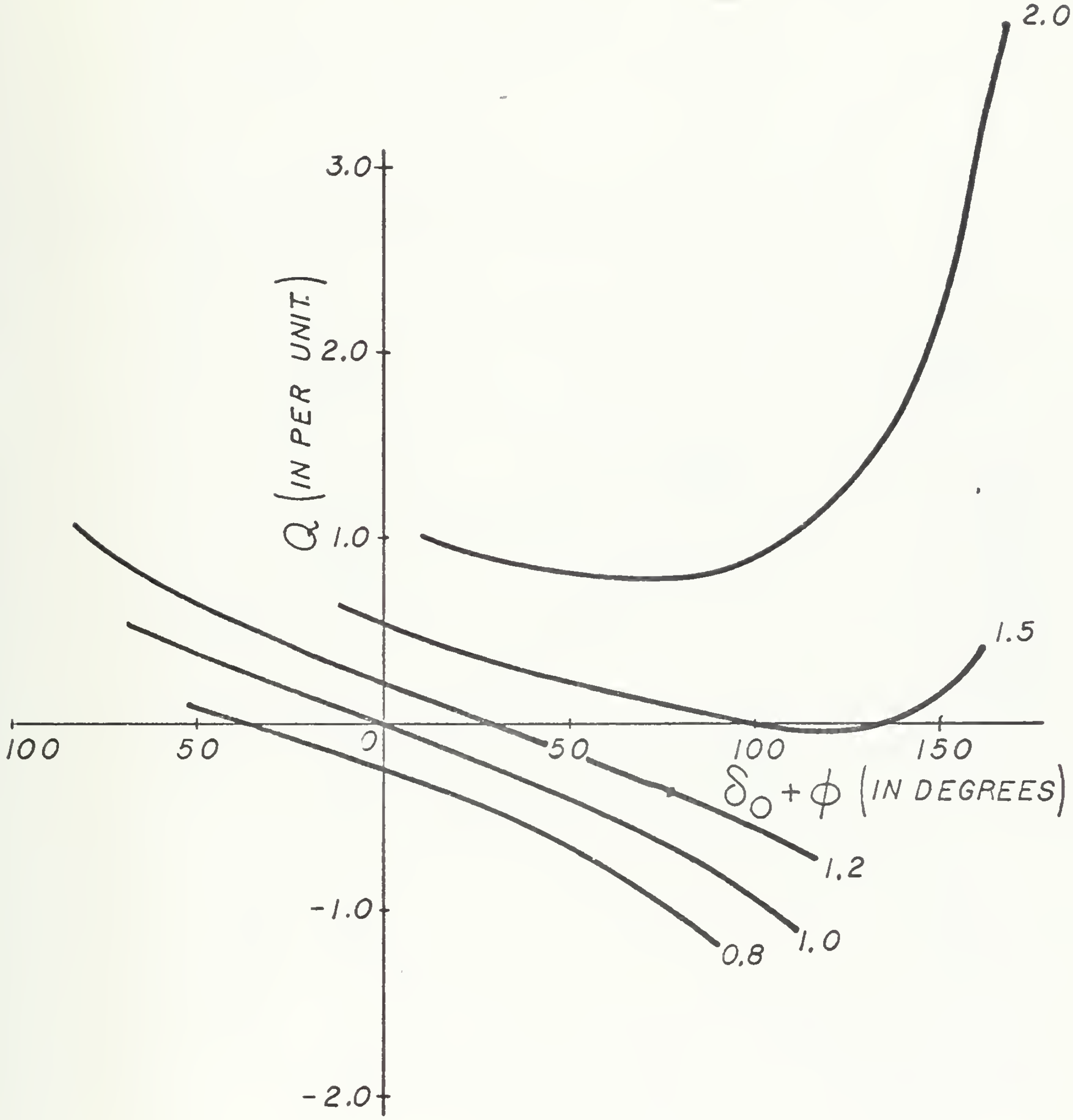

Figure 7

Curves of reactive power, Q, versus power system time phase angle difference, $\delta_o + \phi$, for one per unit real power transfer and various levels of field current, I_f

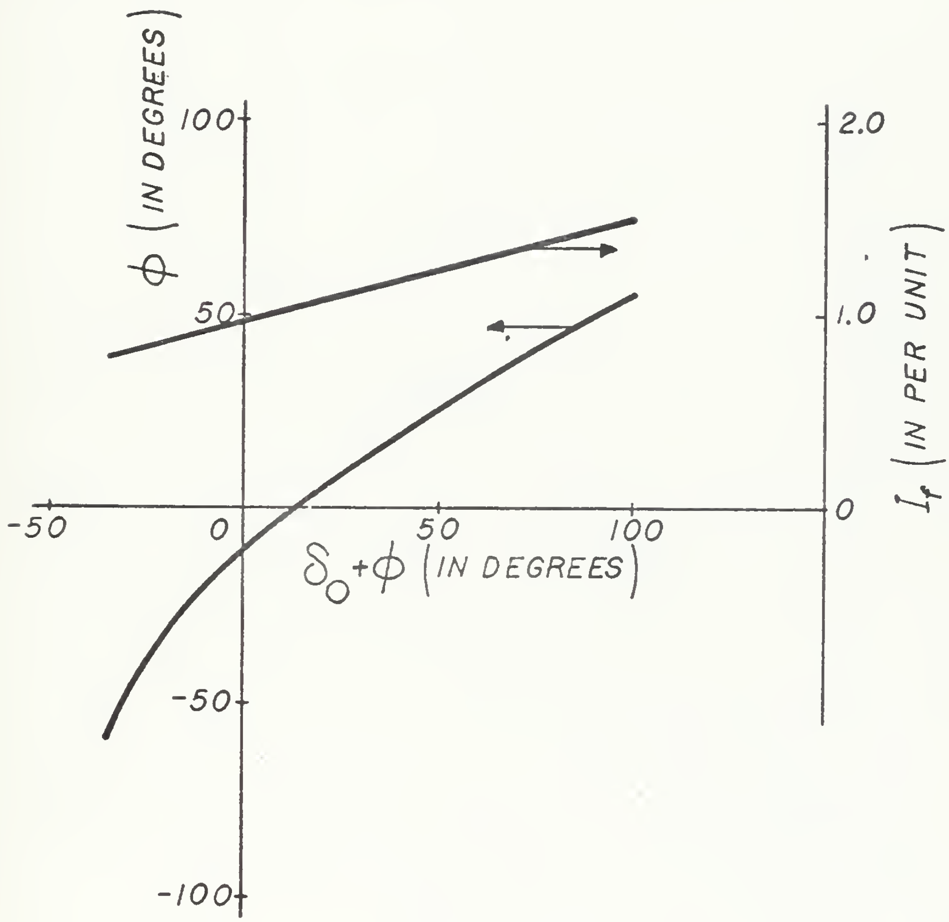

Figure 8

Armature displacement angle, ϕ, and field current, I_f, versus power system time phase angle difference, $\delta_o + \phi$, for one per unit real power transfer and reactive power equal to zero.

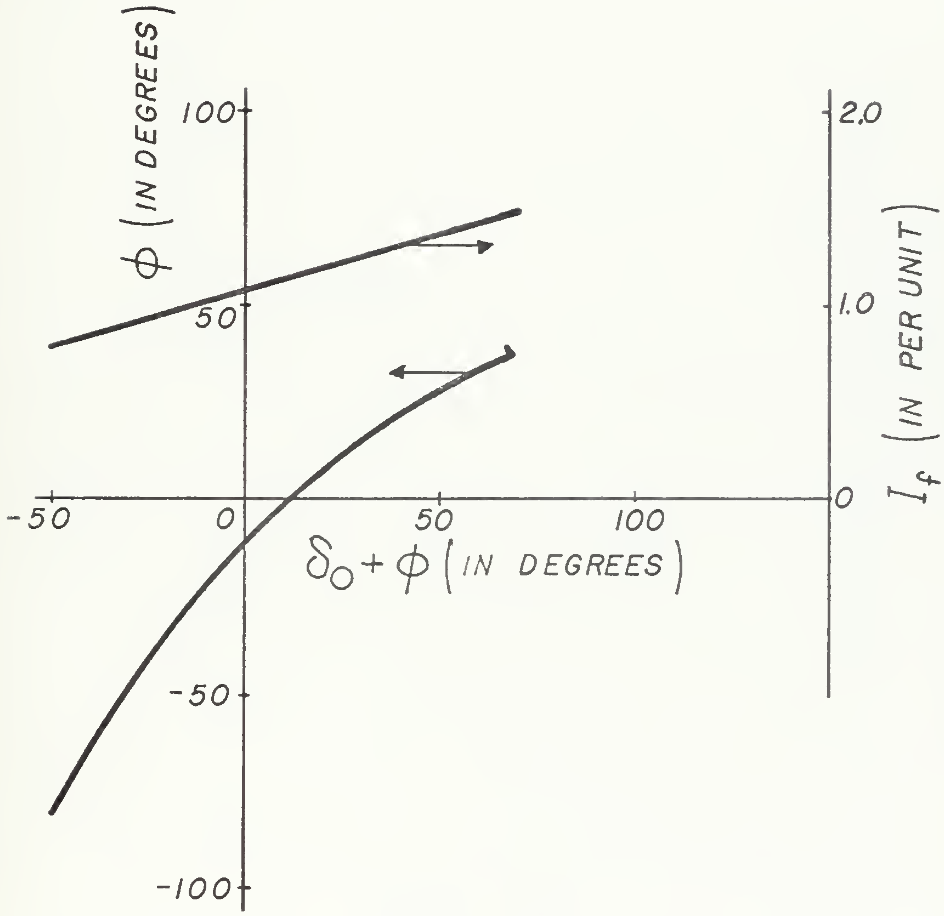

Figure 9

Armature displacement angle, ϕ, and field current, I_f, versus power system time phase angle difference, $\delta_0 + \phi$, for one per unit real power transfer and reactive power equal to + 0.1 per unit.

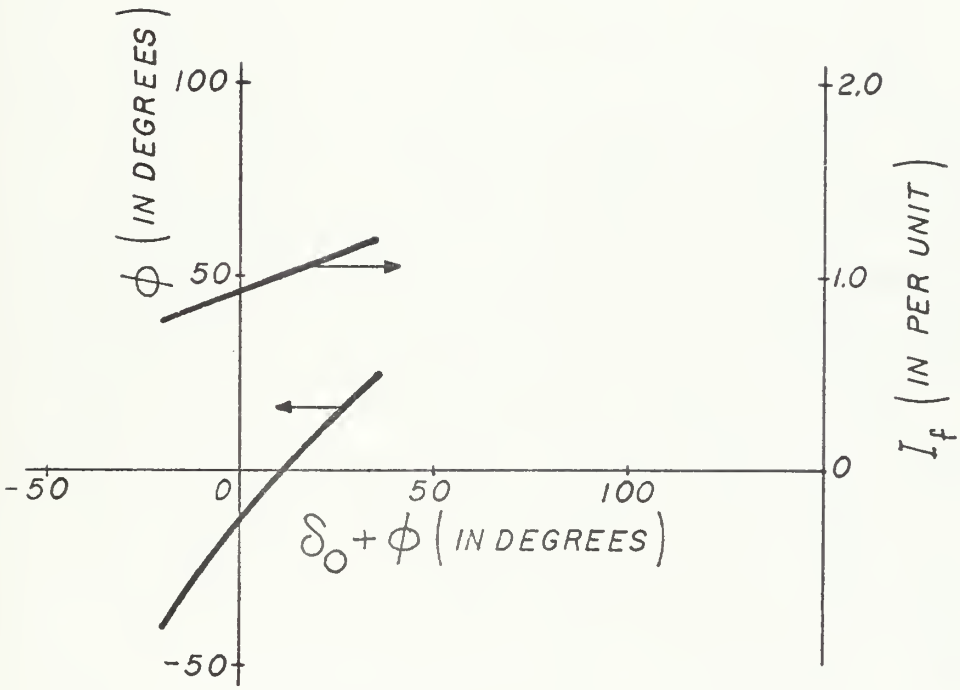

Figure 10

Armature displacement angle, ϕ, and field current, I_f, versus power system time phase angle difference, $\delta_0 + \phi$, for one per unit real power transfer and reactive power equal to -0.1 per unit.

DISCUSSION OF RESULTS

The important result of this thesis is that the synchronous transformer can be characterized by a set of curves in which the armature displacement angle and field current can be determined for a given power system time phase angle difference for operation at real power equal to one per unit and at various levels of reactive power. In an ordinary transformer there is always reactive power flow due to leakage reactance, but in the synchronous transformer settings may readily be chosen for unity power factor power transfer. Additionally, the synchronous transformer can operate at unity power factor when transferring power between two systems that are out of time phase or it can simultaneously act as a transformer and a synchronous condenser between two buses that may be in or out of time phase. Thus it is seen that a synchronous transformer in addition to being able to act as a transformer and as a synchronous condenser, can also act as a phase shifter.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The results of this static analysis have been encouraging. It has been shown that the analytic model of equations (40) through (44) yield a set of curves in which the armature displacement angle and field current can be determined for a given power system time phase angle difference for operation at real power equal to one per unit and at various levels of reactive power. The synchronous transformer can transfer real power from one bus to another whether its time phase is behind, equal to, or ahead of that of the other bus.

Recommendations

It is recommended that further investigations of the synchronous transformer be pursued. Further efforts should include a dynamic analysis with faults and transients and an economic analysis to determine the best level of its usage within the power systems and its general economic feasibility.

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APPENDIX A

TRANSFORMATION TO ROTOR COORDINATES

TRANSFORMATION TO ROTOR COORDINATES

The machine voltage current relationships (equations (10) and (11) may be expressed by equation (A-1) (page 43).

Define a pair of reciprocal transforms relating phase and rotor corrdinates, where u represents a current or a voltage. These transforms are similar to those of Lewis [14].

$$\begin{bmatrix} u_d \\ u_q \\ u_o \end{bmatrix} = T \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \qquad \text{(A-2)}$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = T^{-1} \begin{bmatrix} u_d \\ u_q \\ u_o \end{bmatrix} \qquad \text{(A-3)}$$

$$T \equiv \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \qquad \text{(A-4)}$$

$$T \equiv \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) & 1 \\ \cos(\theta + 2\pi/3) & -\sin(\theta + 2\pi/3) & 1 \end{bmatrix} \qquad \text{(A-5)}$$

By partitioning equation (A-1) as shown we can reduce the notation as shown below and treat the partitions separately.

$$
\begin{bmatrix} V_a \\ V_b \\ V_c \\ \hline V_\alpha \\ V_\beta \\ V_\gamma \\ \hline V_f \end{bmatrix}
= \frac{d}{dt}
\left[\begin{array}{ccc|ccc|c}
L_a & L_{ab} & L_{ab} & M\cos\phi & M\cos(\phi + \frac{2\pi}{3}) & M\cos(\phi - \frac{2\pi}{3}) & L_1 \cos\theta \\
L_{ab} & L_a & L_{ab} & M\cos(\phi - \frac{2\pi}{3}) & M\cos\phi & M\cos(\phi + \frac{2\pi}{3}) & L_1\cos(\theta - \frac{2\pi}{3}) \\
L_{ab} & L_{ab} & L_a & M\cos(\phi + \frac{2\pi}{3}) & M\cos(\phi - \frac{2\pi}{3}) & M\cos\phi & L_1\cos(\theta + \frac{2\pi}{3}) \\
\hdashline
M\cos\phi & M\cos(\phi - \frac{2\pi}{3}) & M\cos(\phi + \frac{2\pi}{3}) & L_\alpha & L_{\alpha\beta} & L_{\alpha\beta} & L_2\cos(\theta - \phi) \\
M\cos(\phi + \frac{2\pi}{3}) & M\cos\phi & M\cos(\phi - \frac{2\pi}{3}) & L_{\alpha\beta} & L_\alpha & L_{\alpha\beta} & L_2\cos(\theta - \phi - \frac{2\pi}{3}) \\
M\cos(\phi - \frac{2\pi}{3}) & M\cos(\phi + \frac{2\pi}{3}) & M\cos\phi & L_{\alpha\beta} & L_{\alpha\beta} & L_\alpha & L_2\cos(\theta - \phi + \frac{2\pi}{3}) \\
\hdashline
L_1 \cos\theta & L_1 \cos(\theta - \frac{2\pi}{3}) & L_1 \cos(\theta + \frac{2\pi}{3}) & L_2 \cos(\theta - \phi) & L_2\cos(\theta - \phi - \frac{2\pi}{3}) & L_2\cos(\theta - \phi + \frac{2\pi}{3}) & L_f
\end{array}\right]
\begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_\alpha \\ i_\beta \\ i_\gamma \\ \hline i_f \end{bmatrix}
\tag{A-1}
$$

$$\begin{bmatrix} V_{P1} \\ V_{P2} \\ V_f \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} A_a & A_{a\alpha} & A_{af} \\ A_{\alpha a} & A_{\alpha} & A_{\alpha f} \\ A_{fa} & A_{f\alpha} & A_f \end{bmatrix} \begin{bmatrix} i_{P1} \\ i_{P2} \\ i_f \end{bmatrix} \tag{A-6}$$

Note that these submatrices are not symmetric, i.e., $A_{\ a} \neq A_{a}$. The off diagonal elements are transposes of their counterparts.

The transformed version of equation (A-6) is:

$$\begin{bmatrix} V_{R1} \\ V_{R2} \\ V_f \end{bmatrix} = \begin{bmatrix} T\frac{d}{dt}A_a T^{-1} & T\frac{d}{dt}A_{a\alpha} T^{-1} & T\frac{d}{dt}\ A_{af} \\ T\frac{d}{dt}A_{\alpha a} T^{-1} & T\frac{d}{dt}\ A_{\alpha} T^{-1} & T\frac{d}{dt}\ A_{\alpha f} \\ \frac{d}{dt}\ A_{fa} T^{-1} & \frac{d}{dt}\ A_{f\alpha} T^{-1} & \frac{d}{dt}\ A_f \end{bmatrix} \begin{bmatrix} i_{R1} \\ i_{R2} \\ i_f \end{bmatrix} \tag{A-7}$$

Carrying out the above matrix operations yields equation (12) of the text.

If the flux-current relationships (equation (10)) are partitioned in the same manner as equation (A-1), they become:

$$\begin{bmatrix} \lambda_{P1} \\ \lambda_{P2} \\ \lambda_f \end{bmatrix} = \begin{bmatrix} A_a & A_{a\alpha} & A_{af} \\ A_{\alpha a} & A_{\alpha} & A_{\alpha f} \\ A_{fa} & A_{f\alpha} & A_f \end{bmatrix} \begin{bmatrix} i_{P1} \\ i_{P2} \\ i_f \end{bmatrix} \tag{A-8}$$

The transformed version of equation (A-8) is:

$$\begin{bmatrix} \lambda_{R1} \\ \lambda_{R2} \\ \lambda_{f} \end{bmatrix} = \begin{bmatrix} T\,A_{a}\,T^{-1} & T\,A_{a\alpha}\,T^{-1} & T\,A_{af} \\ T\,A_{\alpha a}\,T^{-1} & T\,A_{\alpha}\,T^{-1} & T\,A_{\alpha f} \\ A_{fa}\,T^{-1} & A_{f\alpha}\,T^{-1} & A_{f} \end{bmatrix} \begin{bmatrix} i_{R1} \\ i_{R2} \\ i_{f} \end{bmatrix} \tag{A-9}$$

Carrying out this matrix operation yields equation (13) of the text.

APPENDIX B

THE PER-UNIT EQUATIONS

THE PER-UNIT EQUATIONS

There are various techniques for changing equations into per-unit equations, all of them related. The method that I shall employ parallels that of the notes for Power Systems Engineering II, course 6.552 at M.I.T., by Prof. G. L. Wilson.

The following base quantities are specified:

Axis	Base Voltage	Base Current
d	V_o	I_o
q	V_o	I_o
δ	$V_o \frac{N_\alpha}{N_a}$	$I_o \frac{N_a}{N_\alpha}$
ξ	$V_o \frac{N_\alpha}{N_a}$	$I_o \frac{N_a}{N_\alpha}$
f	$V_o \frac{N_f}{N_a}$	$\frac{3}{2} I_o \frac{N_a}{N_f}$

The subscripted N's in the above base quantities represent turns ratios.

The equations to be put into per-unit are the governing equations of the synchronous transformer operating between two buses, equations (34) through (37) and (39). These are listed below:

$$V_1 \cos \delta_a = -\omega(L_E i_q + \frac{3}{2}M \sin\phi \; i_\delta + \frac{3}{2}M \cos\phi \; i_\xi) \tag{B-1}$$

$$V_1 \sin \delta_a = \omega(L_E i_d + \frac{3}{2}M \cos\phi \; i_\delta - \frac{3}{2}M \sin\phi \; i_\xi + \sqrt{\frac{3}{2}} L_1 i_f) \tag{B-2}$$

$$V_2 \cos(\delta_a + \delta_o) = -\omega(-\frac{3}{2}M \sin\phi \; i_d + \frac{3}{2}M \cos\phi \; i_q + L_\varepsilon i_\xi) \tag{B-3}$$

$$V_2 \sin(\delta_a + \delta_o) = \omega(\frac{3}{2}M \cos\phi \; i_d + \frac{3}{2}M \sin\phi \; i_q + L_\varepsilon i_\delta + \sqrt{\frac{3}{2}} L_2 i_f) \tag{B-4}$$

$$0 = \omega L_1 i_q + \omega L_2 i_\xi \tag{B-5}$$

However, we recall that equations (B-1) through (B-4) came from equations (14) through (17), which in steady state per-unit become:

$$(V_d)_{p.u.} = -\frac{\omega}{\omega_o}(\lambda_q)_{p.u.} \tag{B-6}$$

$$(V_q)_{p.u.} = \frac{\omega}{\omega_o}(\lambda_d)_{p.u.} \tag{B-7}$$

$$(V_\delta)_{p.u.} = \frac{\omega}{\omega_o}(\lambda_\xi)_{p.u.} \tag{B-8}$$

$$(V_\xi)_{p.u.} = \frac{\omega}{\omega_o}(\lambda_\delta)_{p.u.} \tag{B-9}$$

Thus we see that the quantities which we must find in per-unit are the fluxes. The flux equation is equation (21), which is broken down into individual flux equations below.

$$\lambda_d = L_E i_\delta + \frac{3}{2} M \cos\phi \; i_\delta - \frac{3}{2} M \sin\phi \; i_\xi + \sqrt{\frac{3}{2}} L_1 i_f \tag{B-10}$$

$$(\lambda_d)_{p.u.} = \frac{\lambda_d}{(V_o/\omega_o)} \tag{B-11}$$

$$(\lambda_d)_{p.u.} = \frac{\omega_o}{V_o} [\ L_E(i_d)_{p.u.}I_o + \frac{3}{2}\ M\ \cos\phi(i_\delta)_{p.u.}I_o(N_a/N_\alpha)$$

$$- \frac{3}{2}\ M\ \sin\phi(i_\xi)_{p.u.}I_o(N_a/N_\alpha) + \sqrt{\frac{3}{2}}\ L_1(i_f)_{p.u.}(\frac{3}{2}\ I_o)(N_a/N_f)\ ] \tag{B-12}$$

$$\lambda_q = L_E i_q + \frac{3}{2}\ M\ \sin\phi\ i_\delta + \frac{3}{2}\ M\ \cos\phi\ i_\xi \tag{B-13}$$

$$(\lambda_q)_{p.u.} = \frac{\lambda_q}{(V_o/\omega_o)} \tag{B-14}$$

$$(\lambda_q)_{p.u.} = \frac{\omega_o}{V_o} [L_E(i_q)_{p.u.}I_o + \frac{3}{2}\ M\ \sin\phi(i_\delta)_{p.u.}I_o(N_a/N_\alpha)$$

$$+ \frac{3}{2}\ M\ \cos\phi(i_\xi)_{p.u.}I_o(N_a/N_\alpha)] \tag{B-15}$$

$$\lambda_\delta = \frac{3}{2}\ M\ \cos\phi\ i_d + \frac{3}{2}\ M\ \sin\phi\ i_q + L_\varepsilon i\ + \sqrt{\frac{3}{2}}\ L_2 i_f \tag{B-16}$$

$$(\lambda_\delta)_{p.u.} = \frac{\lambda_\delta}{(\frac{V_o}{\omega_o})\ (\frac{N_\alpha}{N_a})} \tag{B-17}$$

$$(\lambda_\delta)_{p.u.} = \frac{\omega_o}{V_o}[\frac{3}{2}M\cos\phi(i_d)_{p.u.}I_o\ (N_a/N_\alpha) + \frac{3}{2}M\sin\phi(i_q)_{p.u.}I_o(N_a/N_\alpha)$$

$$+ L_\varepsilon(i_\delta)_{p.u.}I_o(N_a/N_\alpha)^2 + \sqrt{\frac{3}{2}}L_2(i_f)_{p.u.}(\frac{3}{2}I_o)(\frac{N_a^{\ 2}}{N_f N_\alpha})] \tag{B-18}$$

$$\lambda_\xi = -\frac{3}{2}\ M\ \sin\phi i_d + \frac{3}{2}\ M\ \cos\phi i_q + L_\varepsilon i_\xi \tag{B-19}$$

$$(\lambda_\xi)_{p.u.} = \frac{\lambda_\xi}{(\frac{V_o}{\omega_o})(\frac{N_\alpha}{N_a})} \quad \text{(B-20)}$$

$$(\lambda_\xi)_{p.u.} = \frac{\omega_o}{V_o} [- \frac{3}{2} M \sin\phi (i_d)_{p.u.} I_o (N_a/N_\alpha)$$

$$+ \frac{3}{2} M \sin\phi (i_q)_{p.u.} I_o (N_a/N_\alpha)$$

$$+ L_\varepsilon (i_\xi)_{p.u.} I_o (N_a/N_\alpha)^2] \quad \text{(B-21)}$$

Now identify the following quantities:

$$(x_d)_{p.u.} = \frac{\omega_o L_E I_o}{V_o} \quad \text{(B-22)}$$

$$(x_m)_{p.u.} = (\frac{3}{2} \frac{\omega_o M I_o}{V_o}) (\frac{N_a}{N_\alpha}) \quad \text{(B-23)}$$

$$(x_\delta)_{p.u.} = (\frac{\omega_o L_\varepsilon I_o}{V_o}) (\frac{N_a}{N_\alpha})^2 \quad \text{(B-24)}$$

$$(x_{L1})_{p.u.} = \frac{3}{2} \sqrt{\frac{3}{2}} \frac{\omega_o L_1 I_o}{V_o} (\frac{N_a}{N_f}) \quad \text{(B-25)}$$

$$(x_{L2})_{p.u.} = \frac{3}{2} \sqrt{\frac{3}{2}} \frac{\omega_o L_2 I_o}{V_o} (\frac{N_a^2}{N_\alpha N_f}) \quad \text{(B-26)}$$

Now substituting equation (B-22) through (B-26) into the foregoing flux equations and dropping the parentheses and the subscript "p.u." since all quantities are now in per-unit yields:

$$\lambda_d = x_d i_d + x_m i_\delta \cos\phi - x_m i_\xi \sin\phi + x_{L1} i_f \tag{B-27}$$

$$\lambda_q = x_d i_q + x_m i_\delta \sin\phi + x_m i_\xi \cos\phi \tag{B-28}$$

$$\lambda_\delta = x_m i_d \cos\phi + x_m i_q \sin\phi + x_\delta i_\delta + x_{L2} i_f \tag{B-29}$$

$$\lambda_\xi = -x_m i_d \sin\phi + x_m i_q \cos\phi + x_\delta i_\xi \tag{B-30}$$

Substituting these results into equations (B-6) through (B-9) and noting that for steady state synchronous operation $\omega/\omega_o = 1.0$ p.u. yields:

$$V_d = -(x_d i_q + x_m i_\delta \sin\phi + x_m i_\xi \cos\phi) \tag{B-31}$$

$$V_q = x_d i_d + x_m i_\delta \cos\phi - x_m i_\xi \sin\phi + x_{L1} i_f \tag{B-32}$$

$$V_\delta = x_m i_d \sin\phi - x_m i_q \cos\phi - x_\delta i_\xi \tag{B-33}$$

$$V_\xi = x_m i_d \cos\phi + x_m i_q \sin\phi + x_\delta i_\delta + x_{L2} i_f \tag{B-34}$$

Similarly, equation (B-5) can be put into per-unit as follows:

$$0 = \frac{\omega}{\omega_o} L_1 (i_q)_{p.u.}\ I_o + \frac{\omega}{\omega_o} L_2 (i_\xi)_{p.u.}\ I_o (N_a/N_\alpha) \tag{B-35}$$

Multiplication of both terms by the proper constant and assuming steady state operation yields:

$$0 = x_{L1} i_q + x_{L2} i_\xi \tag{B-36}$$

Equations (B-31) through (B-34) and (B-36) are the governing equations for the synchronous transformer in per-unit.

The following definitions are introduced:

$$V_1 = (V_{t1})_{p.u.} \tag{B-37}$$

$$V_2 = (V_{t2})_{p.u.} \tag{B-38}$$

Substitution of the definitions of equations (26) through (29), (B-37), and (B-38), directly yields the per-unit equations for the synchronous transformer operating between two buses.

$$V_1 \sin\delta_a = x_d i_q + x_m i_\delta \sin\phi + x_m i_\xi \cos\phi \tag{B-39}$$

$$V_1 \cos\delta_a = x_d i_d + x_m i_\delta \cos\phi - x_m i_\xi \sin\phi + x_{L1} i_f \tag{B-40}$$

$$V_2 \sin(\delta_a+\delta_o) = -x_m i_d \sin\phi + x_m i_q \cos\phi + x_\delta i_\xi \tag{B-41}$$

$$V_2 \cos(\delta_a+\delta_o) = x_m i_d \cos\phi + x_m i_q \sin\phi + x_\delta i_\delta + x_{L2} i_f \tag{B-42}$$

$$0 = x_{L1} i_q + x_{L2} i_\xi \tag{B-43}$$

APPENDIX C

GAUSS-SEIDEL METHOD

GAUSS-SEIDEL METHOD

The Gauss-Seidel method is a useful technique for solving the following system of simultaneous linear equations:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= a_{1,n+1} \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= a_{2,n+1} \\
\vdots \qquad\qquad\qquad\qquad & \quad \vdots \\
a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n &= a_{n,n+1}
\end{aligned}
\qquad \text{(C-1}
$$

In order to start the algorithm an initial solution vector must be assumed and the interative algorithm is:

$$x_i = \frac{a_{i,n+1}}{a_{ii}} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{a_{ij}}{a_{ii}} x_j \qquad \text{(C-2)}$$

In the iterative algorithm, (C-2), the most recently available values of x_j are always substituted in on the right-hand side.

The convergence criterion is:

$$|x_i(k+1^{st} \text{ iteration}) - x_i(k^{th} \text{ iteration})| < \varepsilon \qquad \text{(C-3)}$$

$$i = 1, 2, \dots, n$$

For computer solution an upper limit on the number of iterations should be specified since convergence may not occur.

Thesis
K289 Kelsey 134539
A static analysis of a
synchronous transformer.